Dimensional analysis refers to the inclusion of dimensions, or units of measurement, in a calculation. Dimensional analysis is important in hydrogeologic problem solving because it helps to ensure that the calculation is done correctly.

Most measurements made in hydrogeology are expressed in the dimensions of length [L] or time [T]. Various combinations of these are possible. For example, measurements of area are expressed in units of length squared, or [L]^2. Discharge measurements are expressed in units of length cubed per time, or [L]^3/[T]. Dimensional analysis essentially means writing the units of measurement at every step of a calculation, canceling them out where appropriate, including conversion factors where appropriate, and checking the units of the final answer to see that they are consistent with the required quantity.

The easiest way to understand dimensional analysis is to examine an example calculation. For this example, let us suppose that we wish to calculate the discharge through a pipe in gallons per minute when the radius of the pipe is known and it is known that water is flowing at a given velocity. For this example, let us assume that the radius of the pipe is 2.0 inches and the velocity of flow is 0.60 foot per second.

The first step in solving this problem using dimensional analysis is to set up the formula for discharge:

\[ Q = V \times A \]

where \( Q \) = discharge, which is expressed in the dimensions [L]/[T]
\( V \) = velocity, which is expressed in the dimensions [L]/[T]
\( A \) = area, which is calculated as \( A = \pi \times \text{(radius)}^2 \) and is expressed in the dimensions [L]^2

The second step is to fill the known values into the formula:

\[ Q = \frac{0.60 \text{ foot}}{\text{second}} \times \pi \times (2.0 \text{ inches})^2 \]

Note that in setting up this equation, it is most helpful to write the quantities by using a horizontal line to indicate division, rather than a slash mark or diagonal line. A little practice with the method will soon make clear that this greatly facilitates the computations.
The third step is to perform the calculation and check to see that the dimensions are appropriate:

\[ Q = \frac{7.5 \text{ feet} \times (\text{inches})^2}{\text{second}} \]

The dimensions are

\[ \frac{[L] \times [L]^2}{[T]} \text{ or } \frac{[L]^3}{[T]} \]

These are the appropriate units for discharge. This tells us that the equation probably has been set up correctly. However, the work is not done yet.

The fourth step is to check to see that the units of the answer are easy to understand and appropriate to the specific situation. In this case, they are not. This is due to two reasons: first, the units are

\[ \frac{\text{feet} \times (\text{inches})^2}{\text{second}} \]

This set of units is not easy to understand in practical terms because it is not internally consistent. But in addition, the problem asks that the discharge be expressed in units of gallons per minute. To achieve this goal, we will have to do some further calculating, using more dimensional analysis.

The next step is to return to the equation with the given values and to do further work on it to yield the desired result. In this case, we must convert the units of feet \(\times\) (inches)\(^2\) to gallons. In addition, we must convert seconds to minutes.

In dimensional analysis, conversions are done by using identities. An identity is a definition of one unit in terms of another. For example, perhaps it is necessary to convert a length measurement of 3 feet into units of inches. The identity that should be used is 1 foot = 12 inches. This identity would be set up in terms of a fraction in which the numerator (or top of the fraction) equals the denominator (or bottom of the fraction). When the numerator equals the denominator, the value of the fraction equals one. And when any quantity is multiplied by one, the result equals the original quantity. In the feet vs. inches example, using dimensional analysis, the conversion would appear this way:

\[ \text{length} = 3 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \]

\[ = 36 \text{ inches} \]

It is important to note a few things in this simple example. First, note that the value of the fraction is one, so the value of the length measurement is not being changed.
by multiplying it by the fraction. Second, note that the answer is expressed in terms of inches only because the units of feet in the numerator and the denominator cancel each other out. This is key because it helps us determine how to write the fraction: We write it so that the unwanted units are canceled out.

Returning now to the original example, in which we attempted to calculate discharge in gallons per minute, we can apply the use of identities in the form of fractions which equal one. Using the identities 1 foot = 12 inches, 1 cubic foot = 7.48 gallons, and 1 minute = 60 seconds, the equation would appear this way:

\[
Q = \frac{7.5 \text{ feet}}{\text{second}} \times \frac{(\text{inches})^2}{1 \text{ foot}} \times \frac{1 \text{ foot}}{12 \text{ inches}} \times \frac{1 \text{ foot}}{12 \text{ inches}} \\
\times \frac{7.48 \text{ gallons}}{1 \text{ foot}^3} \times \frac{60 \text{ seconds}}{1 \text{ minute}}
\]

\[
= \frac{23 \text{ gallons}}{\text{minute}}
\]

Note that the identity 1 foot = 12 inches had to be used twice, because it was necessary to cancel out the units of inches squared (not simply inches).

The final step in dimensional analysis is to check it. Once again, we should check to be sure that the units are appropriate to the quantity being measured. In this case, they are units of \([L^3]/[T]\), which as we have already determined is appropriate for a discharge measurement. In addition, we should check that the units of our answer are the units required in the problem. In this case, the problem asked for units of gallons per minute, which is what we have determined. And finally, we should check the identity factors (conversion factors) to ensure that they have been written correctly. It is surprisingly easy to err by writing an incorrect factor such as "1 inch = 12 feet" or "7.48 ft\(^3\) = 1 gallon" while performing a dimensional analysis.

Following these simple steps will greatly reduce the possibility of errors in computations. In addition, using dimensional analysis makes it simpler to explain to others exactly what has been done in a calculation.
Values of the well function $W(u)$ for Various Values of $u$

Values of the well function, $W(u)$, are calculated based on the following equation:

$$W(u) = -0.5772 - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} + \ldots$$

See the table of calculated values of the well function on the following page.
### Appendix 3  Values of the Well Function

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<th>$W(u)$</th>
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American Society for Testing and Materials. The ASTM publishes numerous standard practices for hydrogeological investigations, many of which are cited by number in this text. Standards are reviewed and revised regularly. A complete list of current standards may be obtained by contacting the ASTM at 100 Barr Harbor Drive, West Conshohocken, Pennsylvania 19428-2959 or by visiting the ASTM World Wide Web site at www.astm.org.


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